International Journal of Theoretical Physics, Vol. 41, No. 7, July 2002 (© 2002)

Fields and States of Hadrons Extended in the Microscopic Finslerian Space-Time

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Received January 22, 2002

From a consideration of extended hadron structure in the microlocal anisotropic Finslerian space-time, the mesonic and baryonic states with their internal quantum numbers such as strangeness, hypercharge, baryon number are constructed. The SU_3 baryonic multiplets of baryons with spin $(j + \frac{1}{2})\,\hbar$ are generated from the SU_3 mesonic multiplets of mesons with spin $j\hbar$. The meson–baryon mass differences are also derived here. The composite particle field of hadrons for the macroscopic space-time are obtained. In particular, the meson field and one particle meson state are considered here. These one particle hadron states of the macroscopic space-time also possess the quantum numbers (strangeness, hypercharge, etc.) which are regarded as the manifestations of the anisotropic nature of the microlocal space-time. The composite fields constructed here are usable in the reduction formulae of the S-matrix approach for strong interaction.

KEY WORDS: hadrons; microscopic Finslerian space-time.

1. INTRODUCTION

Recently, Adler and Santiago (1999) have modified the uncertainty principle from a consideration of the gravitational interaction of the photon and the particle. This is also a standard result of the superstring theory. It follows from this modified gravitational uncertainty principle that there should exist an absolute minimum uncertainty in the position of any particle, and it is of the order of Planck length. Several other authors (Ng and Van Dam, 1994; and references therein) have also discussed the intrinsic limitation to quantum measurements of space-time distances implying an uncertainty of the space-time metric as well as a quantum decoherence for particles heavier than Planck mass. Thus, the space-time can only be defined as averages over local regions and has no meaning locally. This indicates that one should treat the space-time as "quantized" below a fundamental length-scale. Apart from this aspect, the geometry of the space-time

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itself at the microscopic scale might be different from that at the macroscopic scale. In De (1997) the microscopic space-time was considered as an anisotropic Finsler space, and for this space the classical field equation was obtained from a property of the field on the neighboring points of the autoparallel curve. The quantum field has also been derived there for the bispinor field of a free lepton through the quantum generalization of this Finslerian microspace below a fundamental length-scale. In fact, in the process of quantization, the field transforms into a bispinor which can be decomposed as a direct product of the two spinors depending on the position coordinates and the directional arguments, respectively. The former spinor corresponds to the field of the macroscopic spaces (the laboratory Minkowskian space-time, and the large-scale space-time of the universe) which appear as the associated spaces of the Finslerian microspace. This spinor was shown to satisfy the usual Dirac equation with a cosmological time-dependent mass term. The directional variable-dependent spinor for a constituent of the extended hadron-structure in this microlocal space-time, on the other hand, can give rise to an additional quantum number for generating the internal symmetry of hadrons. In De (2001a,b) the electromagnetic field and its interaction with leptons have been introduced in the Finsler space, and by quantum generalization of this space-time the field equations have been obtained. The lepton current and the continuity equations were also considered. The form-invariance of the field equations under the general coordinate transformations of the Finsler space has been discussed there.

In the present article, we form the mesonic and baryonic states and their internal quantum numbers such as strangeness, hypercharge, baryon number, etc. explicitly from the structural aspect of extended hadrons. The hadron fields of the macroscopic space-time are constructed here from their constituent fields of the microlocal space-time. The asymptotic condition for the composite particle field is also considered. The composite fields constructed here for the hadrons can be used in the reduction formulae of the S-matrix approach for the strong interaction.

We begin, in Section 2, with the construction of hadronic states with their internal quantum numbers. The baryonic multiplets having the internal symmetry group SU_3 are shown to generate from the mesonic SU_3 multiplets. The meson–baryon mass differences are also discussed. In Section 3, the hadron fields of the macroscopic space-time are constructed. In particular, the meson field and the one particle meson state are obtained. The one particle hadron states do possess the quantum number like strangeness, baryonic charge, and hypercharge which are actually the manifestations of anisotropic Finslerian character of the microlocal space-time in which the hadrons are extended. In the concluding Section 4, we make some remarks on the earlier works involving two-body nonhypercharge exchange as well as hypercharge exchange reactions in which the field theory with the perturbative technique is applicable and the postulates of the S-matrix approach for the strong interactions are relevant.

2. MESONS AND BARYONS

In the study of the geometry of hadrons it was shown earlier in De (1997) that the internal symmetry algebra generated from the partial reflection group can be made to be the internal symmetry group SU_2 because the constituents in the hadron configuration are in the internal spin-half angular momentum ($s = \frac{1}{2}\hbar$) state. This internal angular momentum was shown to arise from the directional variable-dependent part of the separated bispinor (lepton) field $\psi(\underset{\sim}{x}, \underset{\sim}{\nu})$. Also, the reflection symmetry represented by the fixed values of the third component of internal spin (s_3) for the particle and antiparticle configurations fixes a preferred direction (in the tangent space). Moreover, the internal helicities corresponding to these fixed s_3 values of the constituents can give rise to a conserved quantum number (a conserved "charge"), and are related to the one parameter group U_1. Consequently, one can arrive at the internal SU_3 symmetry group which decomposes into $SU_2 \otimes U_1$. One such a consideration has been made by Bandyopadhyay (1989) with the assumption of an internal $\ell = \frac{1}{2}\hbar$ orbital angular momentum by introducing a preferred direction such that ℓ_3 values $\frac{1}{2}\hbar$ and $-\frac{1}{2}\hbar$ represent particle and antiparticle states. There, the anisotropy is introduced through the magnetic monopole, and in this space the inernal helicity has been connected with the fermion number with only a special choice of the value of a quantum number $\mu = \frac{1}{2}$ where μ denotes the measure of anisotropy. Also, the connection between the internal helicity and the fermion number made by Bandyopadhyay in a complexified space-time is valid only for massless particles because the formula for helicity operator used therein will not hold good when mass is "generated" by the imaginary part of the space-time. With this postulate of half orbital angular momentum for the constituents of a hadron, Bandyopadhyay and Ghosh (1989) have taken a baryonic multiplet corresponding to the internal symmetry group SU_3 representing baryons with spin $\frac{1}{2}\hbar$ to arise from the mesonic SU_3 multiplet (with spin 0) by a spinorial constituent having the symmetry group U_1. As pointed out above, in the present consideration the preferred direction arises in a natural way, and in fact, it is the manifestation of the property of the fields in the Finslerian microscopic space-time of extended hadrons. In this scheme, we get a baryonic state by the introduction of a constituent spinor with a specific internal s_3 value (or a specific internal helicity) in the configuration of a meson. In fact, the two opposite s_3 values for this spinorial constituent correspond to baryons and antibaryons. In mesonic state, the two constituents of it are in the opposite internal helicity states, and consequently bears no signature of internal spin-half angular momentum (or anisotropy of the internal space-time) outside the configuration. Specifically, the baryon number is supposed to be twice the sum of internal s_3 values of the spinorial constituents which are not Majorana spinors. On the other hand, the internal quantum number "strangeness" is the sum of internal s_3 values of the Majorana spinorial constituents in a hadron. Also, we can get the "hypercharge" of a hadron, which is the sum of the baryon number and the strangeness of it. Thus, the additional non-Majorana spinorial constituents appearing in the configuration of a baryon, with fixed internal s_3 values (or internal helicities) are responsible for the baryon number of it. Introduction of a spinor (non-Majorana) with specific internal helicity in the configuration of a meson gives rise to a baryon, and in fact, we can have spin $\frac{1}{2}\hbar$ baryons from spin 0 mesons, spin $\frac{3}{2}\hbar$ baryons from spin one mesons and so on. In general, spin $(j+\frac{1}{2})\hbar$ baryons are constructed from spin $j\hbar$ mesons. As the spinor constituents which are responsible for generating baryonic multiplet (of baryons of spin $(j+\frac{1}{2})\hbar$) having the internal symmetry group SU_3 from the mesonic SU_3 multiplet (of mesons of spin $j\hbar$) have the symmetry group U_1, we can regard this U_1 group to be the baryon number generating group. Thus,

$$SU(3)_{\text{baryons}} \subset U(3) = SU(3)_{\text{mesons}} \otimes U_1 \tag{1}$$

The meson–baryon mass difference can be derived in exactly the same way as done by Bandyopadhyay and Ghosh (1989). The supermultiplets which contain the SU_3 multiplets of mesons and baryons corresponding to spin 0 and $\frac{1}{2}\hbar$ respectively as well as for those of spin $\hbar$ (vector mesons) and spin $\frac{3}{2}\hbar$ baryons in the massless case are being constructed. In breaking this supersymmetry the meson–baryon mass difference is obtained. The symmetry breaking has the group structure

$$U_3 \to SU_2 \otimes U_1 \otimes U_1 \tag{2}$$

This generates the mass difference of the members of different isomultiplets of the SU_3 multiplet as well as a mass difference between the highest massive meson with hypercharge and the lowest massive baryon.

The supermultiplet for pseudoscalor mesons and spin $\frac{1}{2}\hbar$ baryons is taken as

$$\left\{\begin{matrix} \pi^+ & & & & & \sum^+ & & \\ & & K^+ & \bar{K}_0 & p & & & \Xi^0 \\ \pi^0 & \eta & & & & \sum^0 & \Lambda & \\ & & K_0 & K^- & n & & & \Xi^- \\ \pi^- & & & & & \sum^- & & \end{matrix}\right\} \tag{3}$$

The mass splitting of $K-\pi$ as well as those for $Y-N(Y=\Lambda,\sum)$, and $\Xi-Y$ are governed by the decomposition of $SU_3 \to SU_2 \otimes U_1$. The group structure $U_3 \to SU_2 \otimes U_1 \otimes U_1$ relates the mass split between the highest massive meson with the hypercharge, K, and the lowest massive baryon N. Since U_1 group appears twice in this mass splitting, once in the Gell–Mann Okubo type split $SU_3 \to SU_2 \otimes U_1$ and another U_1 for boson fermion distinguishibility, one can have the mass difference as twice the mass difference generated by $SU_3 \to SU_2 \otimes U_1$ split giving rise to $K - \pi$, $Y(\Lambda, \sum) - N$, or $\Xi - Y$ mass difference. Now, if we take the mass difference for $Y - N$ as

$$\frac{3m_{\sum} + m_{\Lambda}}{4} - m_N \simeq 230 \text{ Mev},$$

it is found that

$$m_N - m_K \simeq 2 \times 230 \text{ Mev} = 460 \text{ Mev} \tag{4}$$

which is in agreement with the experimental value. Similarly, one can consider the relation of masses among the vector mesons and spin $\frac{3}{2}\hbar$ decuplet baryons. We can have the following relation

$$m_{\Delta} - m_{K^*} \simeq 2\left(m_{\sum^*} - m_{\Delta}\right) \tag{5}$$

It is also in good agreement with the experimental results. It is to be noted that a supermultiplet is feasible only for the case of massless hadrons, and the breaking of this supermultiplet into meson and baryon multiplets (caused by any mass term which, in fact, destroys the conformal symmetry) suggests that the whole spectra of hadron masses are generated dynamically.

3. HADRON FIELDS AND STATES

We shall now construct hadron fields from the constituent fields of the anisotropic Finslerian microscopic space-time. The constituents of a composite hadronic configuration are situated at the neighboring points of the microdomain. Actually, these constituents are considered to lie on the autoparallel curve whose tangent vectors result from each other by successive infinitesimal parallel displacements. The neighboring points on the autoparallel curve where the constituents lie (or the corresponding fields depend) are (x^μ, ν^μ), $(x^\mu + dx^\mu, \nu^\mu + d\nu^\mu)$, $(x^\mu - dx^\mu, \nu^\mu - d\nu^\mu)$, ..., where dx^μ and $d\nu^\mu$ are quantized, and these differentials are related by (Rund, 1959)

$$d\nu^i = \gamma^i_{hj}(\underset{\sim}{x}, \underset{\sim}{\nu})\nu^h\, dx^j \tag{6}$$

where $\gamma^i_{hj}(\underset{\sim}{x}, \underset{\sim}{\nu})$ are the Christoffel symbols of second kind.

Also, these points are "space-like" separated, that is,

$$\eta_{\mu\nu}\, dx^\mu\, dx^\nu < 0 \tag{7}$$

where $\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1)$.

With the quantum generalization of the space-time we have earlier (De, 1997) obtained the quantum field equations from the equations of classical fields by using a property that the infinitesimal change of a field along the autoparallel curve is proportional to the field itself. For the case of space-like separation this property is expressed as follows:

$$\begin{aligned} \delta\psi(\underset{\sim}{x}, \underset{\sim}{\nu}) &= \psi(\underset{\sim}{x} + d\underset{\sim}{x}, \underset{\sim}{\nu} + d\underset{\sim}{\nu}) - \psi(\underset{\sim}{x}, \underset{\sim}{\nu}) \\ &= (dx^\mu \partial_\mu + d\nu' \partial'_\ell)\, \psi(\underset{\sim}{x}, \underset{\sim}{\nu}) = \frac{\epsilon mc}{\hbar} \psi(\underset{\sim}{x}, \underset{\sim}{\nu}) \end{aligned} \tag{8}$$

where $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$ and $\partial'_\ell \equiv \frac{\partial}{\partial \nu^\ell}$.

Here, m appearing in the constant of proportionality is regarded as the "inherent" mass of the particle (lepton), and ϵ is a real parameter such that $0 < \epsilon \leq \ell$, ℓ being a fundamental length. When the quantization is admitted in two steps, namely,

$$\begin{aligned} &\text{(i)} \quad dx^\mu \to \Delta x^\mu = i\epsilon\gamma^\mu(x), \\ &\text{(ii)} \quad d\nu^\ell \to \Delta\hat{\nu}^\ell = -i\epsilon\gamma^\ell_{h\mu}(\underset{\sim}{x}, \underset{\sim}{\nu})\nu^h\gamma^\mu(x), \text{ (using the relation (6))} \end{aligned} \tag{9}$$

where $\gamma^\mu(\underset{\sim}{x})$ $(\mu = 0, 1, 2, 3)$ are Dirac matrices for the associated curved (Riemannian) space of the Finsler space, we have the quantum field equation for $\psi(\underset{\sim}{x}, \underset{\sim}{\nu})$ which now becomes a bispinor $\psi(\underset{\sim}{x}, \underset{\sim}{\nu}) = \{\psi_{\alpha\beta}(\underset{\sim}{x}, \underset{\sim}{\nu})\}$. The field equation is

$$i\hbar\left\{\gamma^\mu_{\alpha\beta'}(\underset{\sim}{x})\partial_\mu\psi_{\beta'\beta}(\underset{\sim}{x}, \underset{\sim}{\nu}) - \gamma^\mu_{\beta\beta'}(\underset{\sim}{x})\gamma^\ell_{h\mu}(\underset{\sim}{x}, \underset{\sim}{\nu})\nu^h\partial'_\ell\psi_{\alpha\beta'}(\underset{\sim}{x}, \underset{\sim}{\nu})\right\} = mc\psi_{\alpha\beta}(\underset{\sim}{x}, \underset{\sim}{\nu}) \tag{10}$$

The matrices $\gamma^\mu(\underset{\sim}{x})$ are related to the flat space Dirac matrices through the vierbein $V^\mu_\alpha(\underset{\sim}{x})$ as follows:

$$\gamma^\mu(\underset{\sim}{x}) = V^\mu_\alpha(\underset{\sim}{x})\gamma^\alpha \tag{11}$$

Now, for a pseudoscalar meson whose constituents are a lepton and an antilepton, let $\overline{\psi}^{(-)(\alpha,a)}$ and $\psi^{(-)(\beta,b)}$ be the creation parts of the fields for particle (lepton) and its charge conjugate antiparticle, respectively. Here, α and β denote the spin indices for the bispinor, and the indices a and b represent the internal helicities (or the s_3 values). Similarly, the destruction operators for the particle and antiparticle are respectively $\psi^{(+)(\alpha,a)}$ and $\overline{\psi}^{(+)(\beta,b)}$. The field operators $\overline{\psi}^{(-)(\alpha,a)}$ and $\psi^{(-)(\beta,b)}$ are functions of neighboring line support elements $(x^\mu + \frac{1}{2}dx^\mu, \nu^\mu + \frac{1}{2}d\nu^\mu)$ and $(x^\mu - \frac{1}{2}dx^\mu, \nu^\mu - \frac{1}{2}d\nu^\mu)$, respectively, where the differentials dx^μ satisfy (7). Specifically, the field of π^o meson is constructed from the creation parts of the fields of μ^+ and μ^- or from those of ν_μ and $\overline{\nu}_\mu$; that of π^- from the creation parts of the fields of μ^- and ν_μ, and so on. Now, remembering the procedure of quantum generalization in deriving the field equation (10), we can have

$$\psi^A\left(x^\mu \pm \frac{1}{2}dx^\mu, \nu^\mu \pm \frac{1}{2}d\nu^\mu\right)$$
$$= \left\{1 \pm \frac{1}{2} i\epsilon\hbar\left(\gamma^\mu\partial_\mu - \gamma^\ell_{h\mu}(\underset{\sim}{x}, \underset{\sim}{\nu})\nu^h\gamma^\mu\partial'_\ell\right)\right\} \cdot \psi^A(x^\mu, \nu^\mu)$$
$$= \left(1 \pm \frac{1}{2}\epsilon mc\right)\psi^A(x^\mu, \nu^\mu) \tag{12}$$

where $A = (\alpha, a)$ or (β, b), and m is the mass of the constituent. For the neutrino constituents, $m = 0$. Then the creation part of the pion field for the macrodomain is obtained as

$$\phi^{(-)}_M(\underset{\sim}{x}) = C\int d^4\nu\, d\epsilon\, T\chi^\dagger(\underset{\sim}{\nu}, \epsilon)\overline{\psi}^{(-)(\alpha,a)}(\underset{\sim}{x}, \underset{\sim}{\nu})\gamma^5\psi^{(-)(\beta,b)}(\underset{\sim}{x}, \underset{\sim}{\nu})\chi(\underset{\sim}{\nu}, \epsilon) \tag{13}$$

where C is an appropriate normalization factor. Here, we have used the relation (12). The factors $(1 \pm \frac{1}{2}\epsilon mc)$ are absorbed into $\chi(\underset{\sim}{\nu}, \epsilon)$ and $\chi^\dagger(\underset{\sim}{\nu}, \epsilon)$ which are, respectively, a probability density (a spinor or a column vector) and its adjoint. The probability density may depend on the parameter ϵ. This procedure of obtaining the pion field of macroscopic space-time is similar to that of forming a lepton field of this space-time through an "averaging" (De, 1989, 1991, 1997). Here, the time-ordered product is represented by the symbol T.

Similarly, the other hadron fields of the macroscopic space-time can be constructed. For example, if a hadron is composed of $2n$ number of lepton and antilepton constituents (some of them may be Majorana particles) then the creation part of the hadron field is given by

$$\phi^{(-)}_H(\underset{\sim}{x}) = C\int d^4\nu\, d\epsilon\, T\chi^\dagger(\underset{\sim}{\nu}, \epsilon)\overline{\psi}^{(-)(\alpha_1,a_1)}(\underset{\sim}{x}, \underset{\sim}{\nu})\gamma^5\psi^{(-)(\beta_1,b_1)}(\underset{\sim}{x}, \underset{\sim}{\nu})$$
$$\times\chi(\underset{\sim}{\nu}, \epsilon)\chi^\dagger(\underset{\sim}{\nu}, \epsilon)\cdot\overline{\psi}^{(-)(\alpha_2,a_2)}(\underset{\sim}{x}, \underset{\sim}{\nu})\gamma^5\psi^{(-)(\beta_2,b_2)}(\underset{\sim}{x}, \underset{\sim}{\nu})$$
$$\times\chi(\underset{\sim}{\nu}, \epsilon)\chi^\dagger(\underset{\sim}{\nu}, \epsilon)\cdots\overline{\psi}^{(-)(\alpha_n,a_n)}(\underset{\sim}{x}, \underset{\sim}{\nu})\gamma^5\psi^{(-)(\beta_n,b_n)}(\underset{\sim}{x}, \underset{\sim}{\nu})\chi(\underset{\sim}{\nu}, \epsilon) \tag{14}$$

For the case of odd number, $2n + 1$, of constituents, the creation part of the hadron field is given by either

$$\phi^{(-)}_H(\underset{\sim}{x}) = C\int d^4\nu\, d\epsilon\, T\chi^\dagger(\underset{\sim}{\nu}, \epsilon)\overline{\psi}^{(-)(\alpha_1,a_1)}(\underset{\sim}{x}, \underset{\sim}{\nu})\gamma^5\psi^{(-)(\beta_1,b_1)}(\underset{\sim}{x}, \underset{\sim}{\nu})$$
$$\times\chi(\underset{\sim}{\nu}, \epsilon)\chi^\dagger(\underset{\sim}{\nu}, \epsilon)\cdot\overline{\psi}^{(-)(\alpha_2,a_2)}(\underset{\sim}{x}, \underset{\sim}{\nu})\gamma^5\psi^{(-)(\beta_2,b_2)}(\underset{\sim}{x}, \underset{\sim}{\nu})$$
$$\times\chi(\underset{\sim}{\nu}, \epsilon)\chi^\dagger(\underset{\sim}{\nu}, \epsilon)\cdots\overline{\psi}^{(-)(\alpha_n,a_n)}(\underset{\sim}{x}, \underset{\sim}{\nu})\gamma^5\psi^{(-)(\beta_n,b_n)}(\underset{\sim}{x}, \underset{\sim}{\nu})$$
$$\times\chi(\underset{\sim}{\nu}, \epsilon)\cdot\chi^\dagger(\underset{\sim}{\nu}, \epsilon)\overline{\psi}^{(-)(\alpha_{n+1},a_{n+1})}(\underset{\sim}{x}, \underset{\sim}{\nu}) \tag{15a}$$

or

$$\phi_H^{(-)}(\underset{\sim}{x}) = C \int d^4\nu \, d\epsilon \, T \psi^{(-)(\alpha_1,a_1)}(\underset{\sim}{x}, \underset{\sim}{\nu})\chi(\underset{\sim}{\nu}, \epsilon)\chi^{\dagger}(\underset{\sim}{\nu}, \epsilon)\bar{\psi}^{(-)(\alpha_2,a_2)}(\underset{\sim}{x}, \underset{\sim}{\nu})\gamma^5$$
$$\times \psi^{(-)(\beta_2,b_2)}(\underset{\sim}{x}, \underset{\sim}{\nu}) \cdot \chi(\underset{\sim}{\nu}, \epsilon)\chi^{\dagger}(\underset{\sim}{\nu}, \epsilon)\bar{\psi}^{(-)(\alpha_3,a_3)}(\underset{\sim}{x}, \underset{\sim}{\nu})\gamma^5\psi^{(-)(\beta_3,b_3)}(\underset{\sim}{x}, \underset{\sim}{\nu})$$
$$\times \chi(\underset{\sim}{\nu}, \epsilon)\chi^{\dagger}(\underset{\sim}{\nu}, \epsilon) \cdots \bar{\psi}^{(-)(\alpha_{n+1},a_{n+1})}(\underset{\sim}{x}, \underset{\sim}{\nu})\gamma^5$$
$$\times \psi^{(-)(\beta_{n+1},b_{n+1})}(\underset{\sim}{x}, \underset{\sim}{\nu})\chi(\underset{\sim}{\nu}, \epsilon) \tag{15b}$$

In fact, the two fields in (15a) and (15b) correspond to the hadrons which are charge conjugate to each other. In the above expressions for the hadron fields of the macroscopic space-time the fields of the constituents at the neighboring line support elements of the Finsler space are replaced by those at $(\underset{\sim}{x}, \underset{\sim}{\nu})$ by absorbing the factors $(1 \pm \frac{1}{2}\epsilon mcj)$ (j being the positive integers) into the probability density function $\chi(\underset{\sim}{\nu}, \epsilon)$ and its adjoint $\chi^{\dagger}(\underset{\sim}{\nu}, \epsilon)$. Also, for convenience, we have used the same notations $\bar{\psi}^{(-)A}(\underset{\sim}{x}, \underset{\sim}{\nu})$ and $\psi^{(-)A}(\underset{\sim}{x}, \underset{\sim}{\nu})$ for all types of constituents, that is, for charged muons and neutrinos (Dirac or Majorana). We can replace γ^5 in the above expressions for hadron fields by some other matrices according to the transformation properties of the resulting hadron fields in the macroscopic space-time.

Now, we can consider the case of separable field as in De (1997). That is, when the field $\psi(\underset{\sim}{x}, \underset{\sim}{\nu})$ is separated as $\psi(\underset{\sim}{x}, \underset{\sim}{\nu}) = \psi(\underset{\sim}{x})\phi^T(\underset{\sim}{\nu})$ where $\psi(\underset{\sim}{x})$ is the field of the macrodomain which is the Minkowskian space-time, then for the present case we have

$$\bar{\psi}^{(-)(\alpha,a)}(\underset{\sim}{x}, \underset{\sim}{\nu}) = \left(\bar{\phi}^{(a)}(\underset{\sim}{\nu})\right)^T \bar{\psi}^{(-)(\alpha)}(\underset{\sim}{x})$$

or

$$\bar{\psi}^{(-)(\alpha,a)}(\underset{\sim}{x}, \underset{\sim}{\nu}) = \sum_{\vec{p}} \frac{1}{V^{1/2}} \left(\frac{m}{E_{\vec{p}}}\right)^{1/2} c^{\dagger}_{\vec{p},\alpha,a} \bar{\phi}^{(a)^T}(\underset{\sim}{\nu}) \bar{u}^{(\alpha)}(\vec{p}) e^{i\underset{\sim}{p}\cdot\underset{\sim}{x}}, \tag{16}$$

and

$$\psi^{(-)(\beta,b)}(\underset{\sim}{x}, \underset{\sim}{\nu}) = \psi^{(-)(\beta)}(\underset{\sim}{x})\phi^{(b)^T}(\underset{\sim}{\nu})$$

or

$$\psi^{(-)(\beta,b)}(\underset{\sim}{x}, \underset{\sim}{\nu}) = \sum_{\vec{p}'} \frac{1}{V^{1/2}} \left(\frac{m}{E_{\vec{p}'}}\right)^{1/2} d^{\dagger}_{\vec{p}',\beta,b} v^{(\beta)}(\vec{p}')\phi^{(b)^T}(\underset{\sim}{\nu}) e^{i\underset{\sim}{p}'\cdot\underset{\sim}{x}} \tag{17}$$

where the four-momenta $\underset{\sim}{p}$ and $\underset{\sim}{p}'$ are on the mass shell, that is, $p^o = \omega_{\vec{p}} = \sqrt{\vec{p}^2 + m^2}$ and $p'^o = \omega_{\vec{p}'} = \sqrt{\vec{p}'^2 + m^2}$ in the natural unit $c = \hbar = 1$. Here, $c^{\dagger}_{\vec{p},\alpha,a}$ and $d^{\dagger}_{\vec{p}',\beta,b}$ are the creation operators for the particle and antiparticle, respectively.

We are here considering the case of discrete momenta, and for this case the anti-commutation relations are given by

$$\{c_{\vec{p},\alpha,a},\, c^{\dagger}_{\vec{p}',\alpha',a'}\} = \{d_{\vec{p},\alpha,a},\, d^{\dagger}_{\vec{p}',\alpha',a'}\} = \delta_{p\vec{p}'}\delta_{\alpha\alpha'}\delta_{aa'} \tag{18}$$

where $c_{\vec{p},\alpha,a}$ and $d_{\vec{p},\alpha,a}$ are the destruction operators for the particle and antiparticle, respectively. All other anticommutators are equal to 0. For the continuous momenta the summations in (16) and (17) are to be replaced by the integrations, that is, $\frac{1}{V}\sum_{\vec{p}}$ by $\frac{1}{(2\pi)^3}\int d^3p$.

In this case the anticommutation relations (18) are

$$\{c_{\vec{p},\alpha,a},\, c^{\dagger}_{\vec{p}',\alpha',a'}\} = \{d_{\vec{p},\alpha,a},\, d^{\dagger}_{\vec{p}',\alpha',a'}\} = \delta^{(3)}(\vec{p}-\vec{p}')\delta_{\alpha\alpha'}\delta_{aa'} \tag{19}$$

For the following discussion, if we take momentum to be continuous then a problem of normalization regarding the one particle states $c^{\dagger}_{\vec{p},\alpha,a}|0\rangle$ and $d^{\dagger}_{\vec{p},\alpha,a}|0\rangle$ is to be faced with because of the anticommutation relations (19). In fact,

$$\langle 0|c_{\vec{p},\alpha,a},\; c^{\dagger}_{\vec{p},\alpha,a}|0\rangle = \langle 0|\{c_{\vec{p},\alpha,a},\, c^{\dagger}_{\vec{p},\alpha,a}\}|0\rangle = \infty$$

But the problem can be resolved by "smearing" in the momentum space. With a square integrable function $f(\vec{p})$ which is "concentrated" around a peak value one can construct a state as a linear superposition:

$$c^{\dagger}_{f,\alpha,a}|0\rangle = \int d^3p\, f(\vec{p})c^{\dagger}_{\vec{p},\alpha,a}|0\rangle \tag{20}$$

It is to be noted that $\langle 0|c_{f,\alpha,a},\; c^{\dagger}_{f,\alpha,a}|0\rangle$ becomes finite as $f(\vec{p})$ is square integrable. The state $c^{\dagger}_{f,\alpha,a}|0\rangle$ is now interpreted as the one particle state. Keeping in mind all these prescriptions for the continuous momentum case, we now proceed here with the discrete momenta as in (16) and (17). For the constituents with zero masses (that is, for neutrinos) the normalizations in (16) and (17) should be $\frac{1}{V^{1/2}}$ in places of $\frac{1}{V^{1/2}}(\frac{m}{E_{\vec{p}}})^{1/2}$.

Now, with the use of (16) and (17) one can find the hadron fields. For pseudoscalar meson field the creation part of it follows from (13). It is given by

$$\begin{aligned}\phi^{(-)}_M(\underset{\sim}{x}) &= C\sum_{\vec{p},\vec{p}'}\frac{1}{V}\left(\frac{m^2}{E_{\vec{p}}E_{\vec{p}'}}\right)^{1/2} T\overline{u}^{(\alpha)}(\vec{p})\gamma^5 v^{(\beta)}(\vec{p}')\, e^{i(p+p')\cdot x}\\ &\quad\times c^{\dagger}_{\vec{p},\alpha,a}d^{\dagger}_{\vec{p}',\beta,b}\int d^4\nu\, d\epsilon\, \chi^{\dagger}(\underset{\sim}{\nu},\epsilon)\overline{\phi}^{(a)^T}(\underset{\sim}{\nu})\phi^{(b)^T}(\underset{\sim}{\nu})\chi(\underset{\sim}{\nu},\epsilon)\\ &= CT\overline{\psi}^{(-)(\alpha,a)}(\underset{\sim}{x})\gamma^5\psi^{(-)(\beta,b)}(\underset{\sim}{x})G^{(a,b)}(\ell)\end{aligned} \tag{21}$$

where

$$\overline{\psi}^{(-)(\alpha,a)}(\underset{\sim}{x}) = \sum_{\vec{p}}\frac{1}{V^{1/2}}\left(\frac{m}{E_{\vec{p}}}\right)^{1/2} c^{\dagger}_{\vec{p},\alpha,a}\overline{u}^{(\alpha)}(\vec{p})\, e^{i\underset{\sim}{p}\cdot\underset{\sim}{x}} \tag{22}$$

and

$$\psi^{(-)(\beta,b)}(\underset{\sim}{x}) = \sum_{\vec{p}'} \frac{1}{V^{1/2}} \left(\frac{m}{E_{\vec{p}'}}\right)^{1/2} d^{\dagger}_{\vec{p}',\beta,b} V^{(\beta)}(\vec{p}')\, e^{i\underset{\sim}{p}'\cdot\underset{\sim}{x}} \tag{23}$$

are the creation parts of the fields for the constituent particle and antiparticle, respectively, in the macroscopic space-time. $G^{(a,b)}(\ell)$ is given by

$$G^{(a,b)}(\ell) = \int d^4\nu\, d\epsilon\, \chi^{\dagger}(\underset{\sim}{\nu},\epsilon)\bar{\phi}^{(a)^T}(\underset{\sim}{\nu})\phi^{(b)^T}(\underset{\sim}{\nu})\chi(\underset{\sim}{\nu},\epsilon) \tag{24}$$

(since the range of integration for ϵ is from 0 to ℓ)

Similarly, the destruction part of the meson field is found to be

$$\phi_M^{(+)}(\underset{\sim}{x}) = C'T\bar{\psi}^{(+)(\beta,b)}(\underset{\sim}{x})\gamma^5\psi^{(+)(\alpha,a)}(\underset{\sim}{x})G^{(a,b)}(\ell) \tag{25}$$

where $\psi^{(+)(\alpha,a)}(\underset{\sim}{x})$ and $\bar{\psi}^{(+)(\beta,b)}(\underset{\sim}{x})$ are the destruction parts of the fields for the constituent particle and antiparticle, respectively, in the macrodomain. Thus, the meson field $\phi_M(\underset{\sim}{x})$ is given by

$$\phi_M(\underset{\sim}{x}) = \phi_M^{(+)}(\underset{\sim}{x}) + \phi_M^{(-)}(\underset{\sim}{x}) \tag{26}$$

Now, the normalization factors in (21) and (25) can be specified in consistent with the following normalization condition for the meson field:

$$\langle 0|\phi_M(\underset{\sim}{x})|\underset{\sim}{P}\rangle = Z_3^{1/2}\frac{1}{\sqrt{V}}\frac{1}{\sqrt{2\omega_{\vec{p}}}}e^{i\underset{\sim}{P}\cdot\underset{\sim}{x}} \tag{27a}$$

or

$$\langle\underset{\sim}{P}|\phi_M(\underset{\sim}{x})|0\rangle = Z_3^{1/2}\frac{1}{\sqrt{V}}\frac{1}{\sqrt{2\omega_{\vec{p}}}}\, e^{-i\underset{\sim}{P}\cdot\underset{\sim}{x}} \tag{27b}$$

where Z_3 (nonzero) is the wave function renormalization constant. The normalization condition can also be taken as follows:

$$\langle 0|\phi_M(\underset{\sim}{x})|\underset{\sim}{P}\rangle = \frac{1}{\sqrt{V}}\frac{1}{\sqrt{2\omega_{\vec{p}}}}e^{-i\underset{\sim}{P}\cdot\underset{\sim}{x}} \tag{28}$$

These normalization conditions give rise to the specific forms of the factors C and C', and consequently the following expressions for $\phi_M^{(-)}(\underset{\sim}{x})$ and $\phi_M^{(+)}(\underset{\sim}{x})$:

$$\phi_M^{(-)}(\underset{\sim}{x}) = \frac{1}{V^{1/2}}\frac{Z_3^{1/2}}{\sqrt{2\omega_{\vec{p}}}}\frac{T\bar{\psi}^{(-)(\alpha,a)}(\underset{\sim}{x})\gamma^5\psi^{(-)(\beta,b)}(\underset{\sim}{x})G^{a,b}(\ell)}{\langle\underset{\sim}{P}|T\bar{\psi}^{(-)(\alpha,a)}(0)\gamma^5\psi^{(-)(\beta,b)}(0)G^{(a,b)}(\ell)|0\rangle} \tag{29}$$

$$\phi_M^{(+)}(\underset{\sim}{x}) = \frac{1}{V^{1/2}}\frac{Z_3^{1/2}}{\sqrt{2\omega_{\vec{p}}}}\frac{T\bar{\psi}^{(+)(\beta,b)}(\underset{\sim}{x})\gamma^5\psi^{(+)(\alpha,a)}(\underset{\sim}{x})G^{a,b}(\ell)}{\langle 0|T\bar{\psi}^{(+)(\beta,b)}(0)\gamma^5\psi^{(+)(\alpha,a)}(0)G^{(a,b)}(\ell)|\underset{\sim}{P}\rangle} \tag{30}$$

For the normalization condition (28), one has to replace Z_3 in (29) and (30) by unity. Thus, we have

$$\phi_M(\underset{\sim}{x}) = \frac{1}{V^{1/2}}\frac{Z_3^{1/2}}{\sqrt{2\omega_{\vec{p}}}}\left\{\frac{T\bar{\psi}^{(-)(\alpha,a)}(\underset{\sim}{x})\gamma^5\psi^{(-)(\beta,b)}(\underset{\sim}{x})}{\langle \underset{\sim}{P}|T\bar{\psi}^{(-)(\alpha,a)}(0)\gamma^5\psi^{(-)(\beta,b)}(0)|0\rangle} + \frac{T\bar{\psi}^{(+)(\beta,b)}(\underset{\sim}{x})\gamma^5\psi^{(+)(\alpha,a)}(\underset{\sim}{x})}{\langle 0|T\bar{\psi}^{(+)(\beta,b)}(0)\gamma^5\psi^{(+)(\alpha,a)}(0)|\underset{\sim}{P}\rangle}\right\} \quad (31)$$

This can be compared with the pseudoscalar meson field $\phi(\underset{\sim}{x})$ obtained by Haag (1958), Zimmermann (1958), and Nishijima (1958, 1961, 1964) as

$$\phi(\underset{\sim}{x}) = \lim_{\substack{\xi^2>0 \\ \underset{\sim}{\xi}\to 0}} Z_3^{1/2}\frac{1}{\sqrt{V}}\frac{1}{\sqrt{2\omega_{\vec{p}}}}\frac{T\bar{\psi}\left(\underset{\sim}{x}+\frac{1}{2}\underset{\sim}{\xi}\right)\gamma^5\psi\left(\underset{\sim}{x}-\frac{1}{2}\underset{\sim}{\xi}\right)}{\langle 0|T\bar{\psi}\left(\frac{1}{2}\underset{\sim}{\xi}\right)\gamma^5\psi\left(-\frac{1}{2}\underset{\sim}{\xi}\right)|\vec{k}\rangle} \quad (32)$$

The one particle meson state $|\underset{\sim}{P}\rangle$ in the above expressions for $\phi_M^{(+)}(\underset{\sim}{x})$ and $\phi_M^{(-)}(\underset{\sim}{x})$ is, in fact, given by

$$\begin{aligned} |\underset{\sim}{P}\rangle &\equiv c^\dagger_{\vec{p},\alpha,a}d^\dagger_{\vec{p}',\beta,b}|0\rangle \equiv |\vec{p}, \vec{p}', p^o, p'^o, \alpha, \beta, a, b\rangle \\ &= |\vec{p}+\vec{p}', p^o+p'^o, \alpha, \beta, a, b\rangle \end{aligned} \quad (33)$$

That is,

$$|\vec{P}, P^o\rangle = |\vec{p}+\vec{p}', p^o+p'^o, \alpha, \beta, a, b\rangle$$

Here, we have taken the normalization of the state as

$$\langle P \mid \vec{p}, \vec{p}', p^o, p'^o, \alpha, \beta, a, b\rangle = \delta_{\vec{P},\vec{p}+\vec{p}'}\delta_{P^o,p^o+p'^o} \quad (34)$$

For the continuous momenta case, the necessary smearing in the momentum space is to be made. The state $|\underset{\sim}{P}\rangle$ is, thus, the eigenstate of the momentum operator with the eigenvalue $P^\mu = p^\mu + p'^\mu$. This specification of one particle meson state given in (33) ensures the normalization condition (27a,b) which can be verified explicitly by using (22) and (23). This also follows from the property of invariance under infinitesimal space-time translations whose generators are the operators P^μ. In fact, for conservation of P^μ we have

$$\left.\begin{aligned} \bar{\psi}^{(-)(\alpha,a)}(\underset{\sim}{x}) &= e^{i\underset{\sim}{P}\cdot\underset{\sim}{x}}\bar{\psi}^{(-)(\alpha,a)}(0)\,e^{-i\underset{\sim}{P}\cdot\underset{\sim}{x}} \\ \psi^{(-)(\beta,b)}(\underset{\sim}{x}) &= e^{i\underset{\sim}{P}\cdot\underset{\sim}{x}}\psi^{(-)(\beta,b)}(0)\,e^{-i\underset{\sim}{P}\cdot\underset{\sim}{x}} \end{aligned}\right\} \quad (35)$$

Consequently,

$$\begin{aligned} &\langle \underset{\sim}{P}|T\bar{\psi}^{(-)(\alpha,a)}(\underset{\sim}{x})\gamma^5\psi^{(-)(\beta,b)}(\underset{\sim}{x})|0\rangle \\ &= \langle \underset{\sim}{P}|T\,e^{i\underset{\sim}{P}\cdot\underset{\sim}{x}}\bar{\psi}^{(-)(\alpha,a)}(0)\,e^{-i\underset{\sim}{P}\cdot\underset{\sim}{x}}\gamma^5 e^{i\underset{\sim}{P}\cdot\underset{\sim}{x}}\psi^{(-)(\beta,b)}(0)\,e^{-i\underset{\sim}{P}\cdot\underset{\sim}{x}}|0\rangle \\ &= e^{i\underset{\sim}{P}\cdot\underset{\sim}{x}}\langle \underset{\sim}{P}|T\bar{\psi}^{(-)(\alpha,a)}(0)\gamma^5\psi^{(-)(\beta,b)}(0)|0\rangle, \end{aligned} \quad (36)$$

since $|0\rangle$ and $|\underset{\sim}{P}\rangle$ are eigenstates of the operators P^μ with eigenvalues 0 and P^μ respectively. Then, the normalization condition (27b) follows by using (31).

It should be noted that the hadron fields and states as constructed above contain the indices $a, b, \ldots$ representing the internal helicities. These helicities may be regarded as the manifestation of the extended hadron structure (as composite of the constituents) in the anisotropic Finslerian microscopic space-time. The fields and states also contain spin indices $\alpha, \beta, \ldots$, which are responsible for the spin of the hadron. From the indices $a, b, \ldots$, one can have the internal quantum numbers such as strangeness, baryon number, hypercharge, etc. for the hadron. This has been discussed in the previous section. For the meson field or for its one particle state $|\underset{\sim}{P} = |\vec{p}\rangle + \vec{p}', p^o + p'^o, \alpha, \beta, a, b\rangle$, the baryon number as well as strangeness are, in fact, 0 since a and b have equal and opposite values for them.

We have found the pseudoscalar meson field $\phi_M(\underset{\sim}{x})$ in (31). As an example, we can find the neutral scalar field if we take $\overline{\psi}^{(\pm)(\alpha,a)}(\underset{\sim}{x})$ and $\psi^{(\pm)(\beta,b)}(\underset{\sim}{x})$ to correspond the operators (destruction or creation) for the particles μ^+ and μ^- or those for ν_μ and $\overline{\nu}_\mu$. The destruction and creation operators $a^M_{\vec{k}}(t)$ and $a^{M^\dagger}_{\vec{k}}(t)$ of this neutral scalar field as well as the corresponding in and out operators $a^M_{\vec{k}\,\mathrm{in}}$, $a^{M^\dagger}_{\vec{k}\,\mathrm{in}}$, $a^M_{\vec{k}\,\mathrm{out}}$, $a^{M^\dagger}_{\vec{k}\,\mathrm{out}}$ can then be found by the standard procedure. For example, the incoming meson field $\phi^{\mathrm{in}}_M(\underset{\sim}{x})$ is given by

$$\phi^{\mathrm{in}}_M(\underset{\sim}{x}) = \sum_{\vec{k}} \left(a^M_{\vec{k}\,\mathrm{in}}\, f_{\vec{k}}(\vec{x}, t) + a^{M^\dagger}_{\vec{k}\,\mathrm{in}}\, f^*_{\vec{k}}(\vec{x}, t)\right) \tag{37}$$

where $f_{\vec{k}}$ is the solution of Klein–Gordon equation for a mass M, the mass of the meson. These in and out fields are defined with the retarded and advanced functions for this mass M. They are given by (Lurié, 1968)

$$\left.\begin{aligned} \phi^{\mathrm{in}}_M(\underset{\sim}{x}) &= \phi_M(\underset{\sim}{x}) + \int \Delta_R(\underset{\sim}{x} - \underset{\sim}{y}; M)(\Box_y + M^2)\phi_M(\underset{\sim}{y})\, d^4y \\ \phi^{\mathrm{out}}_M(\underset{\sim}{x}) &= \phi_M(\underset{\sim}{x}) + \int \Delta_A(\underset{\sim}{x} - \underset{\sim}{y}; M)(\Box_y + M^2)\phi_M(\underset{\sim}{y})\, d^4y \end{aligned}\right\} \tag{38}$$

Lurié (1968) has discussed how these incoming and outgoing fields could be identified as the correct incoming and outgoing fields for the composite particle (neutral meson). Also, as in the case of elementary particle fields the following asymptotic conditions for the composite particle field:

$$\lim_{t\to-\infty} \langle a|a^M_{\vec{k}}(t)|b\rangle = Z_3^{1/2}\langle a|a^M_{\vec{k}\,\mathrm{in}}|b\rangle \tag{39}$$

and

$$\lim_{t\to+\infty} \langle a|a^M_{\vec{k}}(t)|b\rangle = Z_3^{1/2}\langle a|a^M_{\vec{k}\,\mathrm{out}}|b\rangle \tag{40}$$

remain valid. Here, $|a\rangle$ and $|b\rangle$ are any two normalizable state vectors. In fact, as far as asymptotic conditions and the consequent reduction formulae (that expresses the S-matrix element in terms of time-ordered products of field operators) are concerned there remains no clear distinction between elementary and composite fields. Thus, the composite fields obtained here for the hadrons can be used in the reduction formulae. As the asymptotic conditions and reduction formulae have no relation with the perturbation theory, these can be applied to the strong interactions of hadrons, in which case the usual perturbation technique does not work. Again, since the reduction technique is independent of the detailed form of the Lagrangian, one can use it in the strong interaction in deriving the dispersion relations from some general properties like Lorentz invariance, microscopic causality, etc.

4. CONCLUDING REMARKS

Here we have presented the extended hadron as a composite structure of constituents in the microlocal Finslerian space-time. A baryonic state is obtained by introducing a spinor constituent of specific internal helicity into the configuration of a meson. This internal helicity or the spin-half angular momentum arises for the bispinor field of a lepton constituent in the anisotropic Finslerian microlocal space-time of extended hadrons. It was, in fact, shown in De (1997) that the part of the separable bispinor, which depends on the directional arguments of the Finsler space could be in this internal helicity state, and also that the particle and antiparticle configurations have opposite fixed s_3 values for them. Consequently, the reflection symmetry represented by these fixed s_3 values for the particle and antiparticle constituents gives rise to a preferred direction in the internal space (the tangent space). There we have arrived at the internal SU_3 symmetry group which decomposes into $SU_2 \otimes U_1$. Presently, we have formed the baryonic states and their internal quantum number such as strangeness, hypercharge, baryonic charge, etc. explicitly. Apart from generating baryonic multiplet (of baryons of spin $(j + \frac{1}{2})\hbar$) having internal symmetry group SU_3 from the mesonic SU_3 multiplet (of mesons of spin $j\hbar$), the meson–baryon mass differences are also derived here.

In the present framework of composite hadrons, the hadron fields of the macroscopic space-time have been constructed here from their constituent fields of the microscopic Finslerian space-time. Specifically we constructed the pseudoscalar meson field which was seen to be strikingly similar to that obtained by Haag (1958), Zimmermann (1958), and Nishijima (1958, 1961, 1964). The one particle meson state has also been found here. This state contains the indices which are actually indicating the internal quantum numbers like strangeness, baryon number, hypercharge, etc. The incoming and outgoing meson fields are also obtained by the standard procedure using the retarded and advanced functions. Finally, we arrive at the asymptotic condition for the composite particle field. As pointed out above that the composite fields obtained here for the hadrons can be used in the reduction formulae applicable for the strong interactions of hadrons. Earlier, we (Bandyopadhyay and De, 1973, 1975a,b; De, 1986) have investigated both the nonhypercharge exchange and the hypercharge exchange two-body hadron reactions. The former one is basically dependent on the $\pi\pi$-interaction where the interacting pions are in the structure of the incident hadrons. These pseudoscalar mesons (π^+, π^-, and π^o) are composites of the lepton constituents with particle–antiparticle relation in the configuration of hadron. Phenomenologically one may think of a hadron to be a cluster of pions, with or without a single lepton constituent (muon or neutrino). The strong interaction between the hadrons was regarded as that between the pions in the incident hadron configurations, and consequently the amplitude of this reaction was obtained from the $\pi\pi$-interaction derived from the Lagrangian field theory together with a contribution from the rearrangement of the constituents of the hadrons. The term in the amplitude due to the rearrangement of constituents behaves like $s^{-n\gamma}$ for the large square of the centre-of-mass energy s, where γ is a parameter. This parameter has a correspondence with the Regge amplitude with strongly degenerate trajectories $\alpha(t)$ and residue $\beta(t)$ in the forward regions if one takes $-2\gamma + 1 = \alpha(t)$. Here, n is the number of constituents rearranged. This gives an effective "s-dependent" coupling $g(s) = g \cdot s^{-n\gamma}$, and inserting this factor for each vertex in the perturbation expansion, the final expression can be made convergent because the higher order terms will not be large enough to contribute. Thus, no inconsistency is faced as in the naive form of the field theory. The amplitudes obtained thus were shown to satisfy the axioms of the S-matrix theory such as crossing, analyticity, etc. Also, the amplitude does not violate the Froissart bound at the high energies.

The hypercharge-exchange reactions, on the other hand, were suggested to be dominated by the direct interactions such as knockout or stripping processes which are familiar in nuclear physics (De, 1986). In the version of the rearrangement of the constituents, that is, in the so-called "line physics," these direct interactions can be thought as the conservation of constituent lines in course of the reactions. Although, in this picture, one has to introduce the annihilation of constituents (mesons, muons, or neutrinos) into the vacuum or conversely, the creation of the pair of constituent particle and antiparticle from the vacuum. In fact, in the crossed channel reactions, some constituents annihilate into the vacuum and transfer their four-momenta into the other constituents of the hadrons participating in the reaction. Also, the reverse phenomenon, that is, the generation of a pair of constituents from the vacuum can occur after gaining momenta from the other constituents which are rearranged. As these reactions are dependent on the exchange of a spin-0 (pion) particle, the amplitude was obtained from the one particle exchange model with a modification by the distorted wave Born approximation (DWBA). In general, the one particle exchange models violate the unitarity limit at sufficiently high energy, but in this case of spin-0 exchanged particle this violation does not occur. Also, the term due to the rearrangement of the constituents produces such an s-dependence that ensures the unitarity limit. The DWBA has been introduced to assure both the dip phenomenon where it has been observed experimentally, as well as the unitarity. In these cases of hypercharge exchange two-body reactions the postulates of the *S*-matrix theory were found to be satisfied. Thus, for both types of two-body hadron reactions the dynamics has generated from the structural aspects of the hadrons. The noteworthy fact is that both the field theory in its modified form so that the perturbative expansion remains valid, and the *S*-matrix approach for strong interactions become united in the general premises of the constituents character of the hadron structure extended in the microlocal Finslerian space-time.

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